An Introductory Application of Monte Carlo Simulatoin in Capital Budgeting Analysis

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There is a lack of coverage of an application of Monte Carlo simulation in capital budgeting in introductory level finance textbooks. In more advanced level textbooks, the application is not presented in an easy to understand fashion for students with limited technical background. In this paper, we seek to present an easy application of Monte Carlo simulation in capital budgeting analysis that can be performed using Microsoft Excel alone. In addition, we illustrate that Monte Carlo simulation provides an unbiased estimation of the expected net present value as well as other key input variables. However, we demonstrate that Monte Carlo simulation provides more useful information to a decision maker as compared to conventional scenario or sensitivity analysis. Finally, we provide evidence that, due to the assumption of scenario analysis, the probability of observing the best and worst case NPVs will be very slim. This represents a major weakness of scenario analysis.

INTRODUCTION

The key inputs in a capital budgeting analysis using discounted cash flows (DCF) are the projected future cash flows. These projected cash flows depend on assumptions made regarding forecasted unit sales, unit sales price, variable costs and fixed costs. If these forecasts are inaccurate, the resulting computation of Net Present Value (NPV) will likely be incorrect, leading to a suboptimal capital budgeting decision. Conventional DCF analysis is based on deterministic modeling using single point estimates of expected values of unit sales, sale price, variable cost per unit, and fixed operating costs. However, the expected value is often not easily determinable. To compensate for this inherent uncertainty in the estimation process, the decision-maker might incorporate scenario or sensitivity analysis by asking a series of "what-if" questions. Nevertheless, there is a problem associated with conventional scenario or sensitivity analysis. Even though we have a sense of the sensitivity of the NPV to each scenario or variable, we are still in the dark as to which scenario or value for a given variable is most likely to occur. Monte Carlo simulation is a technique that can overcome the limitations of conventional scenario and sensitivity analysis. Monte Carlo simulation in capital budgeting analysis was first promoted by Hertz (1964) in his seminal article on risk analysis in capital investment.

Monte Carlo simulation is a powerful analytical tool which provides more useful information as compared to conventional sensitivity analysis or scenario analysis. However, there is a general lack of coverage of how to apply Monte Carlo simulation techniques to capital budgeting analysis. Many textbooks geared for undergraduate courses in corporate finance outline sensitivity analysis and scenario analysis in great detail, but all make only cursory or no reference to Monte Carlo simulation. For example, we surveyed three best selling undergraduate corporate finance textbooks written by the following authors: (i) Ross, Westerfield, and Jordan (2009); (ii) Brigham and Houston (2009); (iii) Block, Hirt, and Danielsen (2010). The first two texts explain in great detail the techniques of sensitivity analysis and scenario analysis to measure risk in capital budgeting. However, none of these three books geared to undergraduate students gives a detailed description of the Monte Carlo simulation technique with the help of an example. Alternatively, three popular corporate finance texts geared to graduate students, namely (i) Ehrhardt and Brigham (2008); (ii) Ross, Westerfield, and Jaffe (2009), (iii) Brealey and Myers, and Allen (2010) provide a more detailed but rather complicated discussion of the Monte Carlo simulation approach as applied to capital budgeting. Thus, it appears that there is a very limited coverage of the Monte Carlo simulation approach among undergraduate texts of corporate finance.

Unfortunately, there is also not much written in the academic literature on Monte Carlo simulation in capital budgeting analysis. Golden and Golden (1987) discuss Monte Carlo simulation in capital budgeting analysis using Framework©, a spreadsheet software. While the paper provides the Framework© program module for generating random numbers necessary for the simulation, it fails to explain the simulation process in an easy to understand fashion. Apart from the Golden and Golden (1987) paper, very little has been published on how to introduce and conduct Monte Carlo simulation in capital budgeting analysis with students.

The main goal of this paper is to fill this void by explaining the Monte Carlo simulation approach to capital budgeting in a simple and succinct fashion using just Microsoft Excel. Our rationale for choosing Microsoft Excel to demonstrate Monte Carlo simulation is that it is the most popular business spreadsheet software and is widely available to all business students and practitioners. Also, Microsoft Excel today is sufficiently powerful to handle a simple case of Monte Carlo simulation. Experienced users can also modify Microsoft Excel to handle more complex problems. However, for very complex problems, we suggest that users employ other more sophisticated software such as @Risk© or Oracle Crystal Ball©.

Our demonstration of Monte Carlo simulation in Microsoft Excel is clear and should be easy to understand for readers with limited computer skills and technical background. However, readers with more extensive technical skills should be able to adapt from the framework provided in our paper to handle more complicated business problems or analysis.

We also provide evidence that Monte Carlo simulation provides an unbiased estimation of NPV and other key input variables in addition to providing more useful information as compared to conventional scenario or sensitivity analysis. With the exception of the NPV obtained for best and worst case scenarios, the results from Monte Carlo simulation are not significantly different from those obtained from conventional scenario and sensitivity analyses. However, Monte Carlo simulation provides more useful information as compared to scenario analysis and sensitivity analysis. Specifically, it provides a distribution of expected outcomes while conventional scenario and sensitivity analysis provide only point estimates. A distribution of expected outcomes allows for a more meaningful analysis of the NPV results as compared to point estimates.

Finally, we show that in addition to providing an unbiased estimation of the expected NPV and other key input variables, Monte Carlo simulation also takes into account the interaction (or lack of interaction) among all key input variables. Scenario analysis is not designed to capture interaction among such variables. In this paper, we provide evidence that, due to the assumptions of scenario analysis, the probability of observing the best and worst case NPVs will be very slim. This represents a major weakness of scenario analysis.

The remainder of the paper is organized as follows. The second section explains basic procedures of scenario and sensitivity analysis. The third section describes basic procedures of Monte Carlo simulation. The fourth section compares basic results of Monte Carlo simulation to those derived from conventional scenario and sensitivity analyses. The fifth section demonstrates the accuracy of Monte Carlo simulation. The sixth section discusses advantages of Monte Carlo simulation, while the last concludes the paper.

SCENARIO AND SENSITIVITY ANALYSES

Scenario analysis is a method of studying future events by taking into account multiple possible future outcomes or scenarios. In capital budgeting analysis, scenario analysis commonly focuses on estimating net present value (NPV) of an investment under different scenarios. At a minimum, analysts should focus on: (1) the base case, which is the expected outcome they believe will most likely take place; (2) the best case, which represents the most favorable outcome; and (3) the worst case, which is the least favorable outcome.

When we teach capital budgeting at the undergraduate level, we often focus on four basic variables: the sale price, unit sales, variable cost per unit and fixed costs. The best case scenario is when the sale price and unit sales are both high while the variable cost per unit and fixed costs are both low. The worst case scenario is when the sale price and unit sales are both low while the variable cost per unit and fixed costs are both high.

Sensitivity analysis can be viewed as a subset of scenario analysis. It focuses on the impact on the outcome of changes in just one variable, keeping all other variables unchanged. It is a useful tool in analyzing the impact of forecasting risk surrounding one variable on the overall outcome of the project.

MONTE CARLO SIMULATION

A Monte Carlo simulation is an analytical technique that uses random numbers and probability distribution to study problems involving uncertainty. It has a wide range of applications in science as well as in business. The Monte Carlo method is often used to estimate the expected value or outcome of the problem under consideration. The term "Monte Carlo method" was coined in the 1940s by scientists who worked on nuclear weapons projects in the Los Alamos National Laboratory.[1] Monte Carlo simulation, which is a numerical solution procedure, is often used when it is mathematically difficult or unfeasible to solve for the solution of the problem analytically with equations. For example, consider a function *g(X)* where *X* is a variable with probability density function *f*. We may find the expected value of

$$g(X),\ \bar{g}$$

by evaluating the following integral where subscript *A* denotes all possible values of *X*.

$$\int_A g(x) \int (x) dx = \bar{g} \qquad (1)$$

In reality, it may be mathematically difficult or impossible to derive directly the value of

$$\bar{g}$$

However, the Monte Carlo method can be used to obtain an estimate of

$$\bar{g}$$

by randomly generating the values of *X*, x_i (where *i* ranges from 1 to n), according to the probability density function, *f(X)*. Then, the estimated value of

$$\bar{g},\ \hat{g},$$

can be obtained through:

$$\hat{g} \frac{1}{n} \sum_{i=1}^{n} g(x_i) \qquad (2)$$

If x_i's are properly generated according to probability density function *f(X)*, then

$$\hat{g}$$

should provide an unbiased estimator of

$$\bar{g}$$

according to the law of large numbers. Moreover, we can obtain the variance of the estimate by:

$$\hat{S}^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} [g(x_i) - \hat{g}]^2 \tag{3}$$

In addition, according to the Central Limit Theorem, the distribution of converges to a standardized normal distribution as *n* increases.

$$\frac{\hat{g} - \bar{g}}{\frac{\hat{S}}{\sqrt{n-1}}} \tag{4}$$

Monte Carlo simulation is a powerful analytical tool that has a wide range of applications in finance. There are two major advantages of Monte Carlo simulation. First, it is used when directly solving for the expected outcome is difficult or impossible. For example, it is often used in pricing of complex derivative instruments such as options or debt instruments with embedded options. Second, when there is uncertainty surrounding the final outcome, Monte Carlo simulation also provides an expected range of possible outcomes.[2] In this context, portfolio evaluation and personal financial planning can benefit from Monte Carlo simulation.

In the next section, the Sensitivity Analysis subsection, we will demonstrate an application of Monte Carlo simulation in capital budgeting analysis.

SCENARIO ANALYSIS AND MONTE CARLO SIMULATION COMPARISONS

Project Information

We demonstrate the differences between scenario analysis, sensitivity analysis, and Monte Carlo simulation in capital budgeting analysis with the help of a simple example. Our example is very similar to illustrations that can be found in undergraduate corporate finance textbooks.[3] Assume that a hypothetical firm has the

Table 1. Parameters of Key Input Variables

Variables	Expected (Mean) Value	Standard Deviation
Unit Sales	6,000	303.97
Price per Unit	$80	3.04
Variable Cost per Unit	$60	1.22
Fixed Cost	$50,000	3,039.70

following estimates for its new project. We assume that all four variables are independent and normally distributed with an expected mean and standard deviation as specified in Table 1.[4] We use this example to demonstrate the use of Microsoft Excel in Monte Carlo simulation.

Also, we assume that the initial investment in fixed assets is $200,000. This investment is depreciated using the straight-line method to zero and is projected to be worthless at the end of the project life. The project is expected to last 5 years. The tax rate is 34 percent and the appropriate discount rate for a project of this level of riskiness is 12 percent.

Although we have assumed normal distributions of variables, Microsoft Excel also works well with many different types of probability distributions. In addition, experienced Excel users can modify our example to handle more complex situations, such as inducing correlations between variables. The use of the same random number for more than one variable is one technique to induce correlation among variables. For example, if we use the same random number to generate the sale price and variable cost per unit, then both variables will be correlated.[5] The rationale for using the same random number to generate more than one variable is that these variables are determined by some common economic factors. In this case, we assume that the sale price and variable cost per unit are affected by the same common economic factors such as inflation. We can also assume that one variable is a function of another variable. In this case, after we generate a random number and convert it into a value for one variable, we will use that value to generate another variable. For example, it is reasonable to assume that unit sales are a function of the sale price. In this case, we would randomly generate sale price first and then derive unit sales from the sale price. Then, the sale price and unit sales will be correlated. There are a number of ways to modify Microsoft Excel to handle more complex situations. However, describing all of these techniques is beyond the scope of this paper.

Table 2. Estimated Values Used in Conventional Scenario Analysis

Variables	Expected Outcome	Best Case Scenario	Worst Case Scenario
Unit Sales	6,000	6,500	5,500
Sale Price	$80	$85	$75
Variable Cost per Unit	$60	$58	$62
Fixed Cost	$50,000	$45,000	$55,000

Scenario Analysis

Scenario analysis requires an estimation of the best and worst outcomes of the key input variables. While there is no theory that suggests what the best and worst outcomes should be, Brealey and Myers, and Allen (2010) suggest as an example on page 245 that one way of defining best and worst case scenarios could be to assume that the probability that the actual value will prove to be better (worse) than the best (worst) outcomes is 10 percent. Following this approach, we use a 90 percent confidence interval to define the best and the worst outcomes. We select this confidence interval because it is one of the most common confidence intervals in statistics.[6] Using a 90 percent confidence interval means that the probability of observing a value which is better (worse) than the best (worst) outcome is 5 percent. Therefore, we assume that the best and worst outcomes for key variables are equal to the expected value ± 1.6449*standard deviation of the estimate (95th percentile and 5th percentile). For the best case scenario, the sale price and unit sales will be at the high end or 95th percentile (expected value + 1.6449*standard deviation) and the variable cost per unit and fixed costs will be at the low end or 5th percentile (expected value – 1.6449*standard deviation). For the worst case scenario, the sale price and unit sales will be at the low end or 5th percentile (expected value – 1.6449*standard deviation) and the variable cost per unit and fixed costs will be at the high end or 95th percentile (expected value + 1.6449*standard deviation). Table 2 describes the estimated values used in conventional scenario analysis.

Taking these estimates into consideration, we perform the following scenario analysis in Table 3. Values in parentheses indicate negative numbers.

Sensitivity Analysis

Sensitivity analysis is performed by allowing one variable that is being analyzed to vary while holding all other variables constant. For example, if we would like to perform sensitivity analysis of the sale price, we will hold unit sales, variable cost

Table 3. Results of Conventional Scenario Analysis

Pro Forma Income Statement	Expected Outcome	Best Case Scenario	Worst Case Scenario
Sales	480,000	552,500	412,500
Variable Cost	(360,000)	(377,000)	(341,000)
Fixed Cost	(50,000)	(45,000)	(55,000)
Depreciation	(40,000)	(40,000)	(40,000)
EBIT	30,000	90,500	(23,500)
Tax (34%)	(10,200)	(30,770)	7,990
Net Income	19,800	59,730	(15,510)
OCF	59,800	99,730	24,490
NPV	$15,565.62	$159,504.33	($111,719.03)

Table 4. Results of Conventional Sensitivity Analysis of Sale Price

Pro Forma Income Statement	Expected Outcome	Best Case Scenario	Worst Case Scenario
Sales	480,000	510,000	450,000
Variable Cost	(360,000)	(360,000)	(360,000)
Fixed Cost	(50,000)	(50,000)	(50,000)
Depreciation	(40,000)	(40,000)	(40,000)
EBIT	30,000	60,000	0
Tax (34%)	(10,200)	20,400	0
Net Income	19,800	39,600	0
Operating Cash Flow	59,800	79,600	40,000
NPV	$15,565.62	$86,940.19	($55,808.95)

per unit, and fixed costs constant (at the expected outcome level) and only allow price per unit to vary. Using information in Table 2, we perform sensitivity analysis of a change in the sale price as an example with the expected sale price is $80, best case sale price is $85, and worst case sale price is $75. All other variables are set at the expected outcomes. We illustrate the process of a sensitivity analysis of a sale price in Table 4.

We repeat this sensitivity analysis for each of the other three variables. In Table 5, we report the NPV estimate of sensitivity analyses of the sale price, unit sales, variable cost per unit and fixed costs. Note that the NPV of the expected outcome will be the same for scenario analysis and every case of sensitivity analyses.

Table 5. NPV Estimates from Conventional Sensitivity Analysis of Unit Sales, Sale Price, Variable Cost per Unit, and Fixed Costs

Variables	Expected Outcome	Best Case	Worst Case
Unit Sales	$15,562.62	$39,357.14	($8,225.91)
Price per Unit	$15,562.62	$86,940.19	($55,808.95)
Variable Cost per Unit	$15,562.62	$44,115.44	($12,984.21)
Fixed Costs	$15,562.62	$27,461.38	$3,669.86

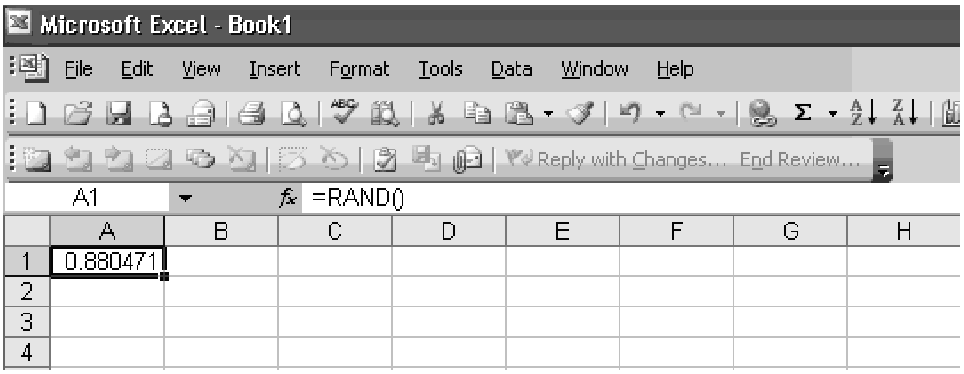
Monte Carlo Simulation

Monte Carlo simulation requires the generation of a random number for each input variable. Next, this random number is converted to a value according to a pre-specified probability distribution function. For this capital budgeting problem, a random number for each key variable (unit sales, sale price, variable cost per unit and fixed cost) is generated and it is then converted to a value according to the probability distribution function for that variable. Then, NPV is computed from these key variables. We repeat this process 1,000 times. Finally, we calculate a mean, the low end of the distribution given by the 5th percentile, the high end of the distribution given by the 95th percentile, standard deviation for each key input variable and NPV. A mean of each key input variable and NPV is used as an estimate of the expected value.

In the past when computing power was relatively scarce and not so powerful, Monte Carlo simulation was difficult to perform as it required the writing of a computer program to generate a series of random numbers and then converting these random numbers into corresponding values according to a pre-specified probability distribution function. Alternatively, users could employ separate computer software such as @RISK© to perform a Monte Carlo simulation. Now, with advancement in computer power and software development, many Monte Carlo simulation problems can be performed using Microsoft Excel alone. We now describe how to perform Monte Carlo simulation for the current capital budgeting problem using only Microsoft Excel.

To perform Monte Carlo simulation in Microsoft Excel, we first need to generate a random number. The command for a random number generation is "=rand( )". When we enter "=rand( )" into a spreadsheet cell, we will get a random number from a uniform distribution whose value is between 0 to 1. In Figure 1, we demonstrate how to generate a random number in Microsoft Excel.

Figure 1. Random Number Generation in Microsoft Excel



The next step is to convert a random number into value according to a variable's probability distribution function. This can be accomplished by solving the equation of an integral of a cumulative distribution function.

Assuming that the variable of our interest is normally distributed with a mean of μ and a standard deviation of σ, we can convert a random number of 0.880471 into a value by solving for *x* in the following equation.

$$0.880471 = \frac{1}{\sigma\sqrt{2\pi}\int_{=-\infty}^{x} e^{\frac{-(t-\mu)^2}{2\sigma^2}}\,dt} \tag{5}$$

While this process may seem daunting to many students, Microsoft Excel has a built-in function to handle this kind of conversion for different types of probability distributions. For example, we can generate a random number and convert it to a value from a normal distribution in one step using the following command:

"=NORMINV(rand(), mu , sigma)"

where "mu" is the mean (expected) value of a variable and "sigma" is the standard deviation of the variable.

In Figure 2, we demonstrate how to generate and convert the random number into unit sales according to the probability distribution defined in Table 1. Note that with this command, we generate a random number and convert this into a value for unit sales.

Figure 2. Conversion of Random Number into Unit Sales

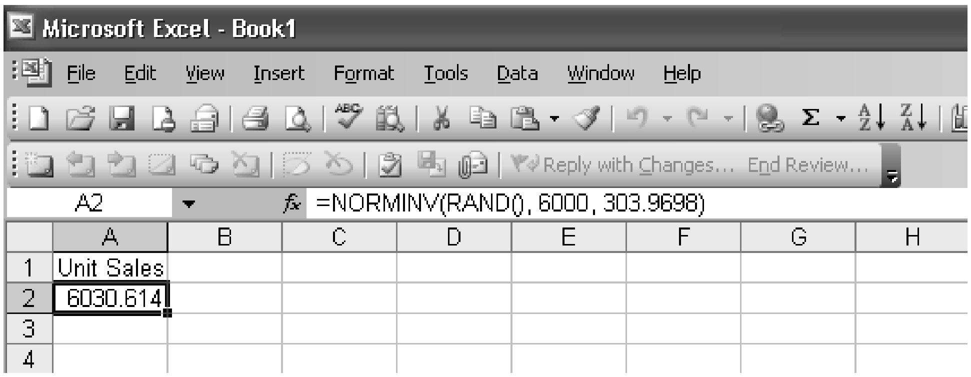
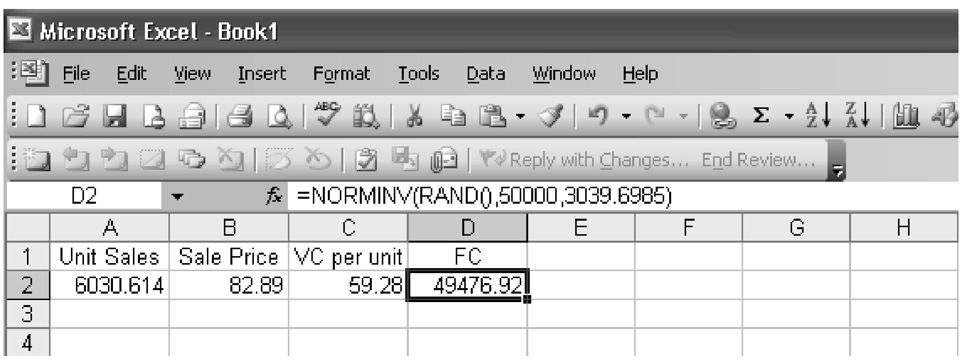


Figure 3. Generating Random Values of All Key Input Variables



Note that Microsoft Excel generates 6030.61 as one random value of unit sales. Next, we can proceed to generate a random value for the sale price, variable cost per unit and fixed costs as follows. In cell B2, we generate a random sale price using the following command "=NORMINV(RAND(), 80, 3.0397". In cell C2, we generate a random variable cost per unit using a following command "=NORMINV(RAND(), 60, 1.2159)". Finally in cell D2, we generate random fixed costs using a following command "=NORMINV(RAND(),50000,3039.6985)". With these commands, we successfully generate random values of all key input variables as shown in Figure 3.

These values will be used to create a pro forma income statement and thereby

Table 6. Comparison of Monte Carlo Based Scenario Analysis and Conventional Scenario Analysis

Variables	Expected Outcome	Worst Case	Best Case	Standard Deviation
Unit Sales	6,003.57	5,518.69	6,494.19	300.75
Price per Unit	$80.07	$75.20	$85.12	$3.05
Variable Cost per Unit	$60.02	$61.99	$58.05	$1.22
Fixed Costs	$50,003.34	$54,851.80	$45,249.60	$2,972.57
NPV	$16,288.16	($57,849.30)	$98,587.80	$48,656.49

to calculate the NPV. This process completes one trial. We can repeat the process as many times as we wish. If we want to have 1,000 trials, the process is repeated 999 more times. Upon completion of 1,000 trials, we will have 1,000 randomly generated values of unit sales, sale price, variable cost per unit and NPV. These values will be used to compute expected values, the distribution of outcomes and other summary statistical values.

Comparing Monte Carlo Based Scenario Analysis to Conventional Scenario Analysis

We report the summary statistics of Monte Carlo based scenario analysis in Table 6 and compare the results to the expected value in Tables 2 and 3. The results of Monte Carlo simulation are based on 1,000 trials. Since we use 90 percent confidence interval to define the best and worst case scenarios, we assign the 95th percentile of the simulated unit sales, sale price and NPV and the 5th percentile of variable cost per unit and fixed costs for the best case scenario. Similarly, we assign the 5th percentile of the simulated unit sales, sale price and NPV and the 95th percentile of variable cost per unit and fixed costs for the worst case scenario. Note that almost all values obtained from Monte Carlo simulation are very close to the values obtained through scenario analysis in Tables 2 and 3 except the best and worst case NPVs. We will compare and contrast the values obtained through Monte Carlo simulation and scenario analysis in Table 8.

Comparing Monte Carlo Based Sensitivity Analysis to Conventional Sensitivity Analysis

To perform Monte Carlo Simulation for sensitivity analysis, we use the same random values that have been generated for scenario analysis. When we perform a Monte Carlo based sensitivity analysis of unit sales, we use the unit sales numbers that have been randomly generated for Table 6 while holding price per unit, variable

Table 7. NPV from Monte Carlo Based Sensitivity Analysis

Sensitivity Analysis of:	NPV of Expected Outcome	NPV of Best Case Scenario	NPV of Worst Case Scenario
Unit Sales	$15,735.59	$39,080.53	($7,336.73)
Price per Unit	$16,499.78	$88,488.9	($53,024.00)
Variable Cost per unit	$15,292.60	$43,413.54	($12,900.54)
Fixed Costs	$15,557.66	$26,867.57	$4,022.34

cost per unit and fixed costs constant at the expected value. We repeat the same process for a Monte Carlo based sensitivity analysis for price per unit, variable cost per unit and fixed costs. We report the NPV results of Monte Carlo based sensitivity analysis in Table 7.

NPVs of different sensitivity analyses obtained through Monte Carlo simulations in Table 7 are close to NPVs obtained through sensitivity analysis in Table 5. We will compare and contrast the values obtained through Monte Carlo simulation and sensitivity analysis in Table 9.

ACCURACY OF MONTE CARLO SIMULATION

A Monte Carlo simulation is a powerful analytical tool since it provides an unbiased estimation of the underlying variables. This is important since any bias or inaccuracy in the estimation of variables can lead to inaccurate NPV estimates. In Table 8, we compare the results of our Monte Carlo based scenario analysis to those of conventional scenario analysis. To test whether the expected (most likely outcome) unit sales, sale price, variable cost per unit, fixed costs and NPV from Monte Carlo simulation are significantly different from the expected values from scenario or sensitivity analysis, we use a standard mean test with a null hypothesis that the mean (expected value) of these values from Monte Carlo simulation are equal to the expected values from scenario analysis.

For best and worst case unit sales, sale price, and NPV, we use 95th percentile of the simulated values as a proxy for best case and 5th percentile as a worst case. For best and worst case variable cost per unit and fixed costs, we use 5th percentile of the simulated values as a proxy for best case and 95th percentile as a worst case. To test whether they are significantly different from the best case and worst case values obtained from scenario and sensitivity analyses, we use the methodology described in Hahn and Meeker (1991).

The results in Table 8 reveal that Monte Carlo simulation provides unbiased estimations for unit sales, price per unit, variable cost per unit, and fixed costs as the differences between theses value generated by Monte Carlo simulation and value

Table 8. Monte Carlo Simulation and Scenario Analysis

Variables	Expected Outcome		Best Case Scenario		Worst Case Scenario	
	Monte Carlo	Scenario Analysis	Monte Carlo	Scenario Analysis	Monte Carlo	Scenario Analysis
Unit Sales	6,003.57	6,000.00	6,494.19	6,500.00	5,518.69	5,500.00
Price	$80.07	$80.00	$85.12	$85.00	$75.20	$75.00
Variable Cost per Unit	$60.02	$60.00	$58.05	$58.00	$61.99	$62.00
Fixed Costs	$50,003.34	$50,000.00	$45,249.60	$45,000.00	$54,851.80	$55,000.00
NPV	$16,288.16	$15,565.62	$98,587.80	$159,504.33***	($57,849.30)	($111,719.03)***

*** Indicate that the difference between the values obtained through scenario analysis and through Monte Carlo Simulation is statistically significant at 1 percent level

** Indicate that the difference between the values obtained through scenario analysis and through Monte Carlo Simulation is statistically significant at 5 percent level

* Indicate that the difference between the values obtained through scenario analysis and through Monte Carlo Simulation is statistically significant at 10 percent level

obtained through scenario analysis are small and not statistically significant at any conventional levels. However when it comes to NPV estimates, Monte Carlo based scenario analysis and conventional scenario analyses provide similar results only in the case of expected outcome. For best and worst case scenarios, the NPV estimates from Monte Carlo simulation and scenario analysis are economically and statistically different. For the best case scenario, NPV from scenario analysis is higher than that from Monte Carlo simulation by $60,916.53. For worst case scenario, NPV from scenario analysis is lower than that from Monte Carlo simulation by $53,869.73. It seems that the NPV spread between the best and worst cases is wider in case of scenario analysis than that of Monte Carlo simulation. From this observation, either conventional scenario analysis overestimates NPV for the best case scenario and negative NPV for the worst case scenario or Monte Carlo simulation under estimates positive NPV for the best case scenario and negative NPV for the worst case scenario.

Upon closer examination, we argue that Monte Carlo based scenario analysis should provide more realistic estimates for the best and worst case scenario NPV as compared to those of conventional scenario analysis. This is because conventional scenario analysis naively assumes that all the best or worst case outcomes will occur concurrently. For example, it assumes that in the best case scenario when the sale price is high, the unit sales will also be high while the variable cost per unit and fixed costs will both be low. Conversely, in the worst case scenario, when the sale price is low, the unit sales will also be low while the variable cost per unit and fixed costs will both be high.

This is an unrealistic and flawed assumption. In the real world, it is unlikely that we will observe the best outcomes (or the worst outcomes) for all variables at the same time. For example, best case scenario analysis implicitly assumes perfectly positive correlation between sale price and unit sales while assume a perfectly negative correlation between sale price and variable cost per unit or fixed costs. In reality, it is quite likely that an increase in the sales price will lead to a drop (rather than an increase) in sales volume, resulting in negative rather than positive correlation. Conversely, a decrease in the sales price might result in higher (not lower) unit sales. When these variables behave independently (as in our example) or are not perfectly correlated, Monte Carlo simulation is able to capture the imperfect correlation among these variables. Scenario analysis cannot capture varying degrees of correlations among variables. It assumes that the best (or worst) case for all variables occur at the same time. We provide evidence that, due to this assumption, the probability of observing the best and worst case NPVs will be very remote. This represents a major weakness of conventional scenario analysis. However, this weakness of scenario analysis has not been discussed in any of the textbooks that we surveyed.

For sensitivity analysis, the correlation (or lack of correlation) among variables is not an issue since we hold all other variables constant and only allow one variable to vary. As we can observe in Table 9, the differences between NPVs generated by

Table 9. Monte Carlo Simulation and Sensitivity Analysis

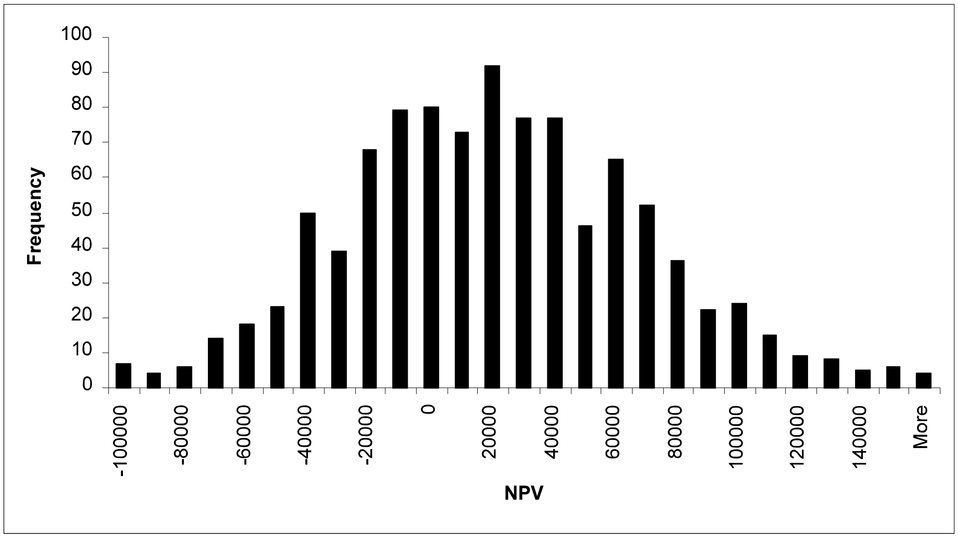
NPV from Sensitivity Analysis of:	Expected Outcome		Best Case Scenario		Worst Case Scenario	
	Monte Carlo	Sensitivity Analysis	Monte Carlo	Sensitivity Analysis	Monte Carlo	Sensitivity Analysis
Unit Sales	$15,735.59	$15,565.62	$39,080.53	$39,357.14	($7,336.73)	($8,225.91)
Price	$16,499.78	$15,565.62	$88,488.90	$86,940.19	($53,024.00)	($55,808.95)
Variable Cost per Unit	$15,292.60	$15,565.62	$43,413.54	$44,115.44	($12,900.54)	($12,984.21)
Fixed Costs	$15,557.66	$15,565.62	$26,867.57	$27,461.38	$4,022.34	$3,669.86

*** Indicate that the difference between the values obtained through sensitivity analysis and through Monte Carlo Simulation is statistically significant at 1 percent level

** Indicate that the difference between the values obtained through sensitivity analysis and through Monte Carlo Simulation is statistically significant at 5 percent level

* Indicate that the difference between the values obtained through sensitivity analysis and through Monte Carlo Simulation is statistically significant at 10 percent level

Figure 4. Distribution of NPVs from Monte Carlo Simulation Based Scenario Analysis



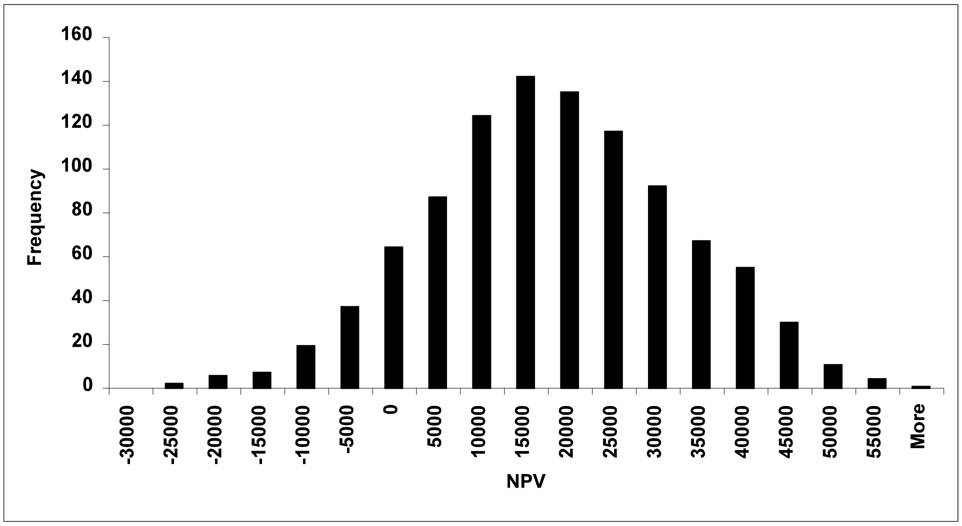
Monte Carlo simulation and those obtained through sensitivity analysis are small and not statistically significant at any conventional levels.

ADVANTAGES OF MONTE CARLO SIMULATION OVER CONVENTIONAL SCENARIO AND SENSITIVITY ANALYSIS

In this section, we will demonstrate that Monte Carlo simulation not only yields unbiased estimation of project NPV but also provides more useful information regarding the distribution of the expected outcomes that cannot be obtained through conventional scenario and sensitivity analysis. Note that conventional scenario and sensitivity analysis provide us with only point estimates of NPV for the expected, best and worst outcomes. However, Monte Carlo simulation provides not only the NPV for the expected, best and worst outcomes, but it also generates a distribution of all expected outcomes.

We present the distribution of NPVs from Monte Carlo based scenario analysis in Figure 4. Since we use 1,000 trials for our Monte Carlo simulation, we will have 1000 simulated values of NPV. The distribution of these values not only provides the expected, best case and worst case NPV, it also yields other useful information such as probability of observing NPVs greater or less than a certain value. One piece of information that should be of interest to a financial manager is how often we observe positive NPVs.

Figure 5. Distribution of NPV from Monte Carlo Simulation Based Sensitivity Analysis of Unit of Sales

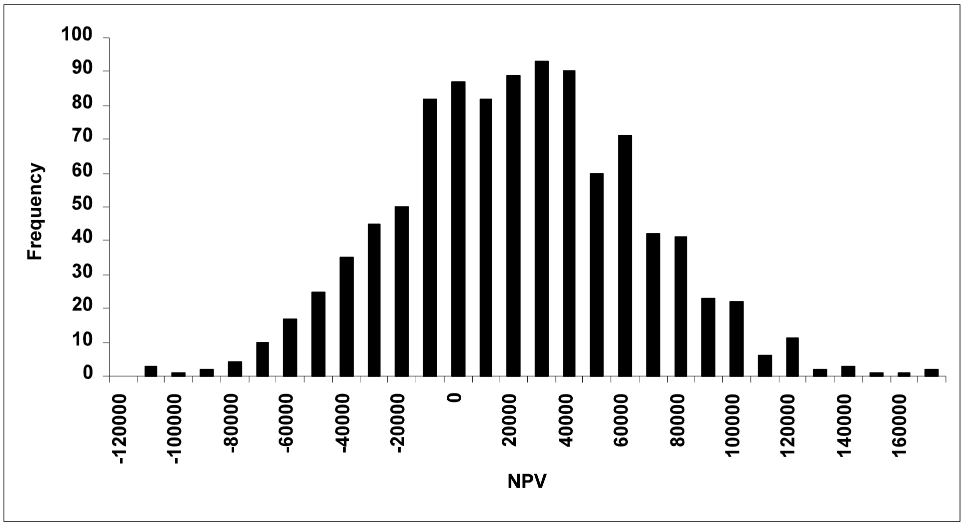


From our simulation results (in an input table that we used to construct the distribution figures but not reported here), we observe positive NPV in 612 trials out of 1000 trials. Hence, we may conclude that in addition to the expected NPV of \$16,288.16, we have approximately 61.20 percent probability of observing a positive NPV. This piece of information can be very critical in decision making as a financial manager may forego an investment with positive expected NPV if there is a high probability that he or she will experience negative NPV.

The distribution of expected NPVs also confirms our conjecture that both the best case scenario NPV and worst case scenario NPV from conventional scenario analysis are unlikely to occur in real life. From our analysis, we observe NPV of greater than or equal to \$159,594.33 (best case scenario NPV from conventional scenario analysis) in only 4 of 1,000 trials (equivalent to 0.4 percent probability) while we observe NPV of less than negative \$111,719.03 (worst case scenario NPV from conventional scenario analysis) in only 3 of 1,000 trials (equivalent to 0.3 percent probability). Hence, we conclude that when all key input variables are not simultaneously at their best or worst forecast levels, as is assumed in conventional scenario analysis, the probability of observing best and worst case scenario NPVs derived from conventional scenario analysis is extremely rare.

In Figures 5 and 6, we report the distribution of NPVs from Monte Carlo based sensitivity analysis of unit sales and sale price to demonstrate the usefulness of

Figure 6. Distribution of NPV from Monte Carlo Simulation Based Sensitivity Analysis of Sale Price



Monte Carlo simulation in sensitivity analysis. For example, Figure 5 shows the sensitivity of NPV to random changes in unit sales keeping all other variables constant. We observe a negative NPV in 135 of 1000 trials, or 13.5 percent of all cases. Similarly, in Figure 6 which measures the sensitivity of NPV to random changes in the unit sales price, we observe a negative NPV in 361 of 1000 trials, or 36.1 percent of all cases. We can therefore see that a negative NPV is more likely to occur due to adverse fluctuations in the unit sale price as compared to overall unit sales.

CONCLUSION

Capital budgeting analysis is one area in Finance that can benefit from the use of Monte Carlo simulation. Monte Carlo simulation is a powerful analytical tool and is relatively easy to apply. However, very little coverage is given to application of Monte Carlo simulations in finance textbooks written for undergraduate students. On the other hand, some graduate level textbooks provide rather complicated discussion of the Monte Carlo simulation approach as applied to capital budgeting. We seek to fill this void in this paper, by demonstrating an application of Monte

Carlo simulation in capital budgeting analysis using nothing more than Microsoft Excel.

We illustrate that Monte Carlo simulation provides an unbiased estimation of NPV and other key input variables. In most cases, the results obtained from Monte Carlo simulation are not significantly different from those obtained from conventional scenario and sensitivity analyses. However, Monte Carlo simulation provides more useful information as compared to conventional scenario and sensitivity analyses. Specifically, Monte Carlo simulation provides a distribution of expected outcomes while scenario and sensitivity analysis provide only point estimates. Knowing the distribution of expected outcomes will enhance the decision making process for financial managers.

Finally, Monte Carlo simulation is able to take into account different degrees of interaction among simulated variables and will therefore provide more realistic estimates of best and worst case NPV. This is because scenario analysis naively assumes that the best (or worst) case for all variables occur at the same time. We provide evidence that, due to this assumption, the probability of observing the best and worst case NPVs will be very remote. This represents a major weakness of conventional scenario analysis which has not been emphasized in finance textbooks that we have surveyed.

ENDNOTES

[1] For details of its applications and its historical background please see Metropolis and Ulam (1949) and Metropolis (1987).

[2] When it is not difficult to directly solve for the expected outcome or there is no uncertainty surrounding the final outcome, Monte Carlo simulation becomes cumbersome and unnecessary.

[3] This example is modified from an example given in Ross, Westerfield, and Jordan (2009). We put in additional assumptions regarding the probability distributions of the key input variables.

[4] While some may argue that our assumption that all four variables are independent and normally distributed is overly simplistic, this example is intentionally kept simple for an undergraduate student audience.

[5] If the probability distribution functions of the two variables are symmetric and of the same type, using the same random numbers to generate two variables will create a perfectly positive correlation between the two end results. If the probability distribution functions of the two variables are not symmetric or not of the same type, the degree of correlation will depend on the types of the probability functions. To induce a negative correlation, we can use one random number, x, for one variable and use (1-x) for another variable. If we desire less than perfect correlation between two variables, we may introduce an error term into one of the variables. In this case the degree of correlation will depend on the probability distribution of the error term: the higher the variance, the lower the correlation between two variables.

[6] Changing the confidence interval will change the values that represent the best and worst outcomes but will not affect the general conclusion of this study.

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